Maximum Lifetime Routing in Wireless Ad Hoc networks I - Theory

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- System model
- Maximum life-time routing problem
- Iterative algorithm
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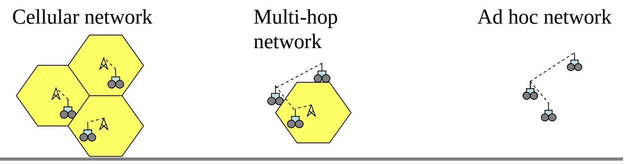
- Effect of power sensitive topology
- Effect of SIR to rate mapping
- Conclusions

Wireless Ad Hoc Networks

- Some of the transceivers (nodes) act as routers
- The network is formed by means of self-configuration without any pre-existing infrastructure
- Network control is distributed among the nodes, no central control unit is needed

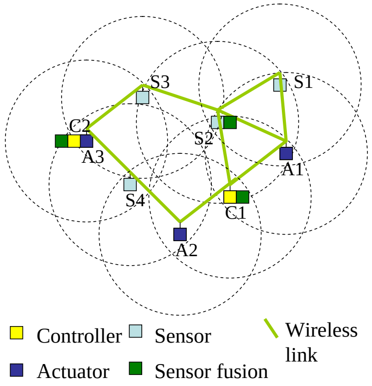


Wireless Ad Hoc Networks

Radio resource management in interference limited ad hoc networks (Academy of Finland)

- Research problems
  - Accurate modeling of co-channel interference
  - Power control, topology control
  - Data rate control, link adaptation
  - Routing
  - Multi-hop 'virtual' cellular networks

Sensor and actuator networks



ERHE Project (TEKES ÄLY)

- Communication
  - Radio technology
  - Medium access
  - Networking and routing
  - Security
- Sensor fusion
  - Filtering sensor data
  - Detecting sensor faults
  - Sensor calibration
- Control
  - Compensation of communication delays
  - Coordination of control loops (hierarchical control)

Ad Hoc Networks

- Benefits
  - No need for separate base stations or the number of them could at least be decreased
  - Easy to deploy, no wiring required
  - Reconfigurable: Network can quickly adapt to topology changes
  - Can be utilized in an areas where infrastructure doesn’t exists
  - Robust: Break down of single network node doesn’t prevent networking
- Drawbacks
  - Distributed operation => difficult to control
  - Lower capacity, higher packet delay and more severe jitter than in cellular/infrastructure networks
  - Nodes are either battery operated or use energy scavenging => Utilized protocols should be energy efficient
  - Network maintenance can be expensive (e.g. change of batteries etc.) => Redundancy is required

Sensor network

- Sensor network is an ad hoc network, in which the nodes are sensor devices
  - Low computation power, small memory space
  - Energy consumption is critical: If nodes are running on batteries their operation time depends heavily on the energy efficiency of the data transmission.
  - Node size and cost are critical: Nodes should be cheap enough, so that they could be deployed in large quantities to obtain both coverage and redundancy
  - Data rates are small: Size of single measurement data unit is only few bytes; data rates 1 … 100 kbytes per second
  - Number of nodes is large: Ad hoc data network ~10 nodes, sensor network ~1000 nodes.

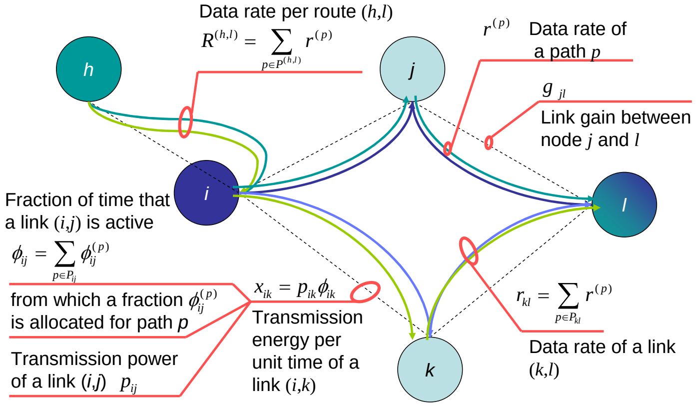
Applications

- Mobile ad hoc networks (MANETS)
  - Ambient intelligence
  - Military communications
  - Personal area networks
  - Peer-to-peer networking: CSCW, mobile gaming etc.
  - Multi-hop extensions to cellular systems
- Wireless sensor and actuator networks
  - Ambient intelligence
  - Military sensor networks
  - Healthcare and emergency response units
  - Logistics
  - Automation and control
  - Agriculture and forestry
  - Research

Objective

- Maximize the operation time of the worst node in the network by jointly controlling
    - Transmit power
    - Scheduling
    - Data rate
    - Routing

System model (1/3)



Data rate per route (h,l)

$$R^{(h,l)} = \sum_{p \in P^{(h,l)}} r^{(p)}$$

$r^{(p)}$ Data rate of a path p

g_{jl} Link gain between node j and l

Fraction of time that a link (i,j) is active

$$\phi_{ij} = \sum_{p \in P_{ij}} \phi_{ij}^{(p)}$$

from which a fraction $\phi_{ij}^{(p)}$ is allocated for path p

Transmission power of a link (i,j) p_{ij}

$$x_{ik} = p_{ik}\phi_{ik}$$

Transmission energy per unit time of a link (i,k)

$$r_{kl} = \sum_{p \in P_{kl}} r^{(p)}$$

Data rate of a link (k,l)

System model (2/3)

DSSS waveforms

- Instantaneous data rate

$$r_{ij}(t) = \frac{W}{\Gamma_{ij}} \frac{g_{ij}(t)p_{ij}(t)}{\sum_{\substack{k\neq i\\k\neq j}} \left[g_{kj}(t) \sum_{\substack{l\neq k\\l\neq i\\l\neq j}} p_{kl}(t) \right] + \nu_j}$$

  - Transmitter power constraint

$$p_{ij}(t) = \begin{cases} p_{ij} & \text{if the link } (i,\, j) \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

$$0 \leq p_{ij} \leq p^{max}$$

  - Peak rate constraint

$$0 \leq r_{ij}(t) \leq r^{max}$$

Snapshot assumption

- Average data rate

$$r_{ij} = \phi_{ij} \min \left\{ r_{max}, \frac{W}{\Gamma} \frac{g_{ij}p_{ij}}{\sum_{\substack{k\neq i\\k\neq j}} g_{kj} \sum_{\substack{l\neq k\\l\neq i\\l\neq j}} \phi_{kl}p_{kl} + \nu_j} \right\}$$

  - Transmission time constraint

$$\phi_{ij} \geq \frac{r_{ij}}{r_{max}} \overset{def}{=} \phi_{ij}^{min}$$

υ_j Noise power
W Chip rate
Γ_{ij} SIR-target

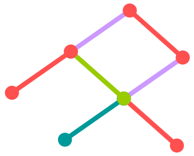
System model (3/3)

- Node cannot transmit and receive simultaneously

$$\sum_j \left(\sum_{p \in \mathcal{P}_{ij}} \phi_{ij}^{(p)} + \sum_{p \in \mathcal{P}_{ji}} \phi_{ji}^{(p)} \right) \leq 1$$

- "Edge coloring" cost

$$\sum_j \left(\sum_{p \in \mathcal{P}_{ij}} \phi_{ij}^{(p)} + \sum_{p \in \mathcal{P}_{ji}} \phi_{ji}^{(p)} \right) \leq \delta < 1$$



Maximum lifetime routing problem (1/2)

- Lifetime of a node i

$$\tau_i = \frac{E_i}{\sum_{j\neq i}\phi_{ij}(P_t + p_{ij}) + \sum_{j\neq i}\phi_{ji}P_r + P_s}$$

E_i	Battery energy
P_t	Transmitter base band power consumption
p_{ij}	Transmitter RF-power
P_r	Receiver power consumption
P_s	Terminal CPU power consumption

- Objective function

$$f(\phi, p) = \min_i\{\tau_i\} \Leftrightarrow F(\phi, p) = \max_i\left\{\frac{1}{\tau_i}\right\}$$

Maximum lifetime routing problem (2/2)

$$\min_{\{\phi,p\}} \max_i\left\{\frac{1}{E_i}\left(\sum_{j\neq i}\left(\sum_{p\in P_{ij}}\phi_{ij}^{(p)}\left(P_t + p_{ij}\right) + \sum_{p\in P_{ij}}\phi_{ji}^{(p)}P_r\right) + P_s\right)\right\}$$

Subject to
- Data rate constraints

$$r^{(p)} = \frac{W}{\Gamma^{(p)}}\frac{g_{ij}p_{ij}\phi_{ij}^{(p)}}{\sum_{\substack{k\neq i\\k\neq j}} g_{kj}\left[\sum_{\substack{l\neq k\\l\neq i\\l\neq j}}\sum_{q\in\mathcal{P}_{kl}} p_{kl}\phi_{kl}^{(q)}\right] + \nu_j} \qquad \sum_{p\in P^{(i,j)}} r^{(p)} \geq R^{(i,j)}$$

- Transmission time constraints

$$\sum_j\left(\sum_{p\in\mathcal{P}_{ij}}\phi_{ij}^{(p)} + \sum_{p\in\mathcal{P}_{ji}}\phi_{ji}^{(p)}\right) \leq 1 \qquad \phi_{ij}^{(p)} \geq 0 \qquad \phi_{ij} = \sum_{p\in P_{ij}}\phi_{ij}^{(p)} \geq \phi_{ij}^{min}$$

- Transmission power constraint

$$0 \leq p_{ij} \leq p_{\max}$$

Transmission energy allocation

- For a given rate to path allocation $R = \left(r^{(p)}\right)$, the transmission energy can be written as

$$x_{ij} = \frac{\sum_{q\in\mathcal{P}_{ij}} r^{(q)}\Gamma^{(q)}}{W}\left(\sum_{\substack{k\neq i\\k\neq j}}\sum_{\substack{l\neq k\\l\neq i\\l\neq j}}\frac{g_{kj}}{g_{ij}}x_{kl} + \frac{\nu_j}{g_{ij}}\right), \quad x_{ij} = p_{ij}\phi_{ij}$$

- By ordering the links, the energy can be solved using a simple Jacobi iteration (distributed power control algorithm)

$$x_i^{(n+1)} = \xi_i(R)\left(\sum_j h_{ij}x_j^{(n)} + \eta_i\right) \overset{def}{=} I_i\left(R, X^{(n)}\right), \quad X = \left(x_{a_ib_i}\right)$$

$$\xi_i(R) = \frac{\sum_{q\in\mathcal{P}_{a_ib_i}} r^{(q)}\Gamma^{(q)}}{W}$$

Joint energy, time, and power allocation

- Power and peak rate constraints

$$p_{ij} \leq p^{max} \Leftrightarrow \phi_{ij} \geq \frac{x_{ij}}{p^{max}}$$

$$r_{ij}(t) \leq r^{max} \Rightarrow \phi_{ij} \geq \phi_{ij}^{min}$$

- JETP algorithm

$$\begin{aligned} x_i^{(n+1)} &= \mathcal{I}_i(R, X^{(n)}) \\ \phi_i^{(n+1)} &= \max\left\{\frac{x_i^{(n+1)}}{p^{max}}, \phi_i^{min}\right\} \\ p_i^{(n+1)} &= \frac{x_i^{(n+1)}}{\phi_i^{(n+1)}} \end{aligned}$$

can be shown to be converge for any feasible rate $R = \left(r^{(p)}\right)$ allocation from any nonnegative initial value $X^{(0)} \geq 0$
(Standard interference function in the sense of Yates)

Transmission rate allocation

- For fixed energy, time, and power allocation, the objective function can be written as

$$F(X,P)=\max_i\left\{\frac{1}{E_i}\left(\sum_{j\neq i}\left(\left(1+\frac{P_t}{p_{ij}}\right)x_{ij}+\frac{P_r}{p_{ij}}x_{ji}\right)+P_s\right)\right\}$$

By noting that

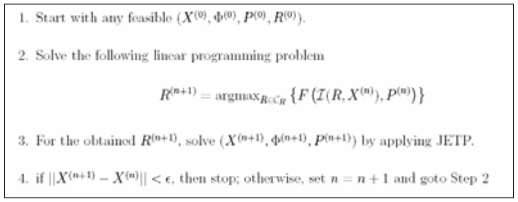
$$x_{ij}=\frac{\sum_{q\in\mathcal{P}_{ij}} r^{(q)}\Gamma^{(q)}}{W}\left(\sum_{\substack{k\neq i\\k\neq j}}\sum_{\substack{l\neq k\\l\neq i\\l\neq j}}\frac{g_{kj}}{g_{ij}}x_{kl}+\frac{\nu_j}{g_{ij}}\right)$$

is a linear function of $R=\left(r^{(p)}\right)$

=> Rate can be solved using linear programming

Iterative routing algorithm

- IR algorithm



1. Start with any feasible $(X^{(0)},\Phi^{(0)},P^{(0)},R^{(0)})$.
2. Solve the following linear programming problem

$$R^{(n+1)}=\text{argmax}_{R\in\mathcal{C}_R}\left\{F\left(\mathcal{I}(R,X^{(n)}),P^{(n)}\right)\right\}$$

3. For the obtained $R^{(n+1)}$, solve $(X^{(n+1)},\Phi^{(n+1)},P^{(n+1)})$ by applying JETP.
4. if $||X^{(n+1)}-X^{(n)}||<\epsilon$, then stop; otherwise, set $n=n+1$ and goto Step 2

The above algorithm is a special case of nonlinear Gauss-Seidel algorithm. Since $F(X,P)$ can be shown to be concave for one of the variables at the time, the iteration is convergent.

Numerical example (1/4)

- System parameters

$W=1$ MHz

$\Gamma=5$ dB

$\upsilon_i=-174$ dBm/Hz

$P_t=800$ mW

2 mW $\leq p_{ij}\leq 1000$ mW

$P_r=400$ mW

$P_s=50$ mW

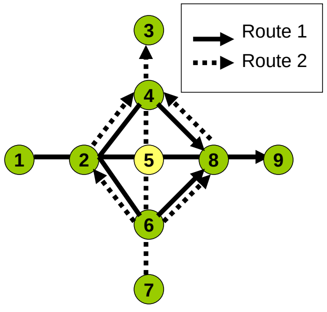
$E_i=E, i\neq 5 \quad E_5=0.1E$

User data rate:

$R^{(1,9)}=R^{(7,3)}=30$ kbit/s

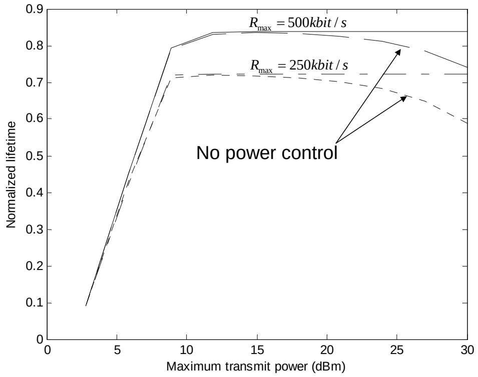
Signalling data rate:

$R^{(i,j)}=1$ kbit/s

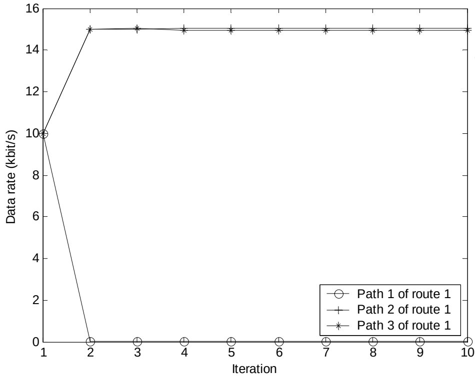


Numerical example (2/4)

E_5=0.1E



Numerical example (3/4)

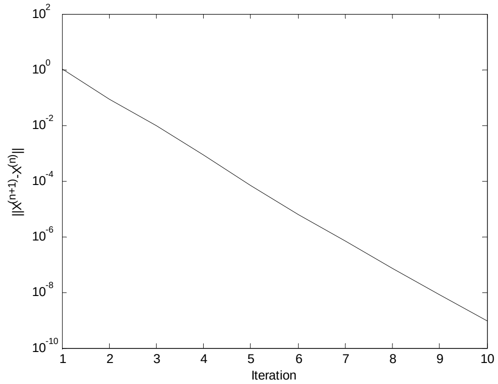


15.0358 kbit/s

14.9642 kbit/s

0 kbit/s

Numerical example (4/4)



E_5=0.9E

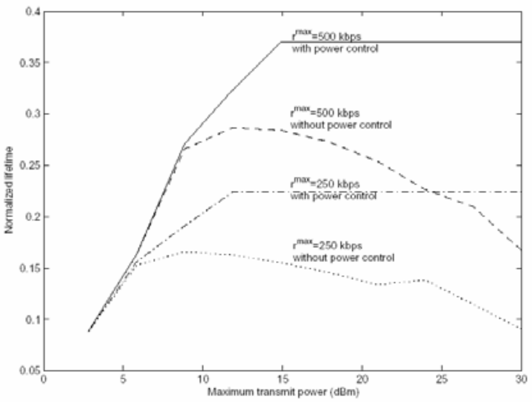


Figure 4: Normalized network lifetime
Normalized by $\min_i\{E_i\}/P_s$

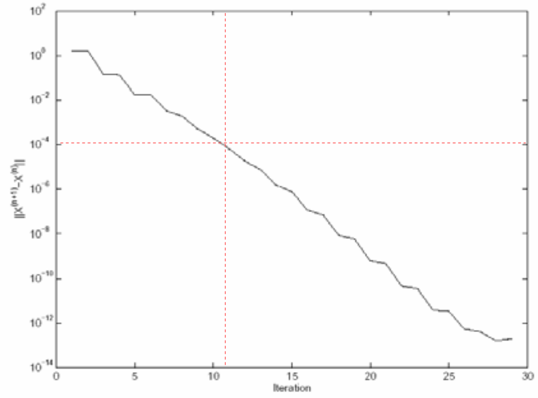


Figure 2: Convergence of X as a function of iterations

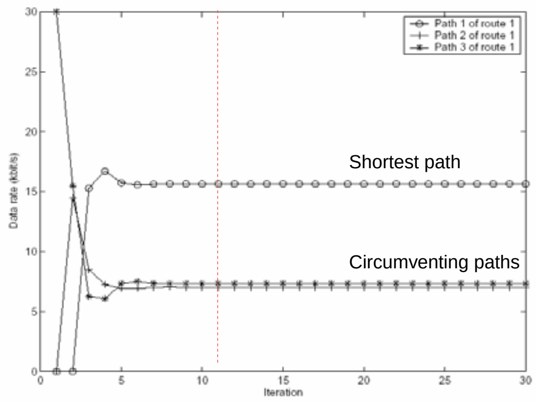


Figure 3: Rate per path allocation as a function of iterations

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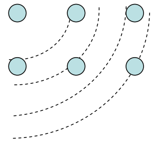
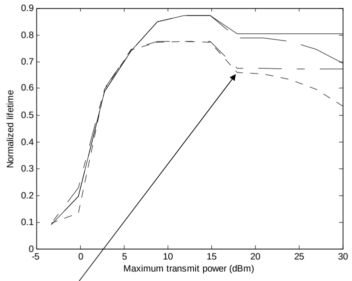
Power dependent topology

- Grid network



  - When power increases more links become available.
  - The longer the link, the higher the interference it causes and the lower the peak data rate.
  - Signaling over the long links consume considerable amount of resources even if no data is allocated to such links.



Signalling over the long links consume a lot of power even if they are not used for data transmission

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Logarithmic rate relation

- Data rate of a link

$$r_i = \phi_i W \log_2 \left(1 + \frac{1}{\sum_j h_{ij}\phi_j + \frac{\eta_i}{p_{max}}}\right)$$

- Rate control mapping

$$\begin{aligned} \phi_i^{(n+1)} &= \zeta_i(R)\Psi\left(\sum_j h_{ij}\phi_j + \frac{\eta_i}{p_{max}}\right) \\ \zeta_i(R) &= \frac{\sum_{q\in\mathcal{P}_{a_jb_j}} r^{(q)}}{W} \\ \Psi(x) &= \frac{1}{\log_2(1+x^{-1})} \end{aligned}$$

$p_i = p_{max} \quad \forall i$ Fixed transmission power

Logarithmic rate relation

- For sufficiently small R the mapping

$$J_i(R,\phi) = \zeta_i(R)\Psi\left(\sum_j h_{ij}\phi_j + \frac{\eta_i}{p_{max}}\right)$$

is contraction.

  - It follows from the mean-value theorem that

$$|J_i(R,\phi) - J_i(R,\phi')| \le \zeta_i(R)\Delta\left(\sum_j h_{ij}\phi_j - \phi'_j\right)$$

$$\Delta = \frac{d}{dx}\Psi(x)|_{x=x^*} = \Psi^2(x^*)\frac{x^*}{x^*(x^*+1)}$$

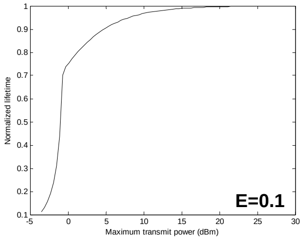
$$x^* \in \left[\sum_j h_ij\phi_j + \frac{\eta_i}{p_{max}}, \sum_j h_ij\phi'_j + \frac{\eta_i}{p_{max}}\right]$$

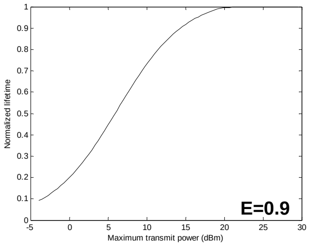
  - Let J=(J_i) and Z(R)=diag(ζ_i(R)). It can be shown that there exists nonnegative diagonal weight matrix W such that

$$||J(R,\phi) - J(R,\phi')||_\infty^W \le \Delta\rho(Z(R)H)||\phi - \phi'||_\infty^W$$

Numerical example

- In both cases, the algorithm converges solutions that circumvent node 5.





Concluding remarks

- Our analysis suggest that if the rate is a linear function of the instantaneous SIR, then the lifetime of the worst node is a non-decreasing function of the maximum air interface power p^{max}.
- If link adaptation is used without power control, i.e. by setting $p_{ij} = p^{max} \quad \forall i, j$, then there exists a threshold value after which any excess power affects the lifetime negatively.
- The routing problem can be solved using a simple iterative algorithm.
  - Energy, time, and power can be solved in a distributed fashion for each link separately using a scheme similar to traditional distributed power control
  - Rate allocation (routing) is based on linear programming and must be solved in centralized fashion.